Test 1 Practice, Math 1410

Answers

I have included the answers below, but not complete worked out solutions. If you have questions about how to get an answer, just ask.

1) A particle moves along a horizontal line so that after t seconds its position is given by

$$s(t) = \frac{5}{3}t^3 - 10t^2 + 15t$$

When is the particle moving left? Note: positive direction is to the right, so increasing = moving right. (Also, you should use the derivative rules to solve this question, you do not have to use the definition of the derivative).

The particle is moving left on the interval $(1, 3)$

2) Sketch the graph of the continuous function $f(x)$ on $[-3, 3]$ that satisfies:

$f(1) = -2, f(-1) = 0$

$f'(1) = 0$, and $f'(-2)$ does not exist.

$f'(x) > 0$ on the intervals (-3, -2) and (1,3).

$f'(x) < 0$ on (-2,1)

$f''(x) > 0$ on (-3, -2) and (-2,3)

3) True of False: If true, explain why it is true, if false give a counterexample or explain why it is false.

(i) If $x = 1$ is a vertical asymptote of $y = f(x)$, then f is undefined at $x = 1$.

False. According to the definition, if $x = 1$ is a vertical asymptote then $\lim_{x \to 1^+} f(x) = \pm\infty$

or $\lim_{x\to 1^-} f(x) = \pm\infty$. Usually f will be undefined there as well, but not necessarily. A counterexample is:

(ii)

$$\lim_{x\to 1} \frac{x^2+6x-7}{x^2+5x-6} = \frac{\lim_{x\to 1} x^2+6x-7}{\lim_{x\to 1} x^2+5x-6}$$

False. The left-hand side simplifies to 8/7, and the right-hand side is 0/0 which is an indeterminate form.

4) Sketch a graph of $f'(x)$ if $f(x)$ is the graph given below:

5) Use the limit definition of the derivative to calculate $\frac{d}{dx}(\sqrt{x-7})$.

$\frac{d}{dx}(\sqrt{x-7}) = \lim_{h\to 0} \frac{1}{h}(\sqrt{x+h-7} - \sqrt{x-7}) = \lim_{h\to 0} \frac{1}{h} \frac{x+h-7-(x-7)}{\sqrt{x+h-7}+\sqrt{x-7}} = \lim_{h\to 0} \frac{1}{\sqrt{x+h-7}+\sqrt{x-7}} = \frac{1}{2\sqrt{x-7}}$

6) Find a value for c such that the limit exists:

$$\lim_{x\to 3} \frac{x^2-7x+c}{x-3}$$

$c = 12$